4-th Nordic Mathematical Contest

April 5, 1990

1. Let m, n, and p be positive odd integers. Show that $\sum_{k=1}^{(n-1)^p} k^m$ is divisible by n.

2. Prove that if $a_1, a_2, \dots, a_n$ are real numbers, then
$$\sqrt[3]{a_1^3 + a_2^3 + \cdots + a_n^3} \leq \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2}.$$

3. A point P is given inside a triangle ABC. A line l through P not containing A intersects AB and AC at Q and R, respectively. Find the line l such that the perimeter of $\triangle AQR$ is least possible.

4. With any positive integer n, three operations f, g, h are allowed: $f(n) = 10n$, $g(n) = 10n + 4$, and $h(2n) = n$. Prove that starting from number 4, every positive integer can be obtained by performing finitely many operations f, g, h in some order.

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